Cincinnati preprint October.1994

Renormalizability and Quantum Stability of the Phase Transition in Rigid String Coupled to Kalb-Ramond Fields II

M. Awada*
Physics Department
University of Cincinnati,Cincinnati, OH-45221

Abstract

Recently we have shown that a phase transition occurs in the leading approximation of the large N limit in rigid strings coupled to long range Kalb-Ramond interactions. The disordered phase is essentially the Nambu-Goto-Polyakov string theory while the ordered phase is a new theory. In this part II letter we study the first sub-leading quantum corrections we started in I. We derive the renormalized mass gap equation and obtain the renormalized critical line of the interacting theory. Our main and final result is that the phase transition does indeed survive quantum fluctuations.

* E-Mail address: moustafa@physunc.phy.uc.edu

Polyakov [1] has argued that the string theory appropriate to QCD should be one with long range correlations of the unit normal. The Nambu-Goto (NG) string theory is not the correct candidate for large N QCD as it disagrees with it at short distances. The NG string does not give rise to the parton-like behaviour observed in deep inelastic scattering at very high energies. The observed scattering amplitudes have a power fall-off behaviour contrary to the exponential fall-off behaviour of the NG string scattering amplitudes at short distances. The absence of scale and the power law behaviour at short distances suggest that the QCD string must have long range order at very high energies. Pursuing this end, Polyakov considered modifying the Nambu action by the renormalizable scale invariant curvature squared term (rigid strings). The theory closely resembles the two dimensional sigma model where the unit normals correspond to the sigma fields. In the large N approximation, there is no phase transition. Polyakov suggested adding a topological term to produce a phase transition to a region of long range order. In [2], we coupled the rigid strings instead to long range Kalb-Ramond fields. Since spin systems in two dimensions may exhibit a phase transition with the inclusion of long range interactions, it is natural to conjecture likewise for rigid strings with long range Kalb-Ramond fields. Indeed we proved that there is a phase transition to a region of long range order in the large N approximation. Such a theory may therefore be relevant to QCD.

Our proof in [2] of the phase transition was based on the leading order approximation in large N, where N is the space-time dimensions. Even though, the large N limit is a successful approximation for non-linear sigma models, and some spin systems it can sometimes lead to an incorrect conclusion. The leading order of the large N approximation is mean field theory which can give incorrect predictions in lower dimensions. For example mean field theory incorrectly gives a phase transition in the one dimensional Ising model. This discrepancy is resolved by carefully examining the sub-leading quantum corrections (loops) where one shows that such quantum corrections in fact destroy the phase transition. Therefore it is crucial to examine the quantum loop corrections to the mass gap, and the critical line of our model of rigid string coupled to long range Kalb-Ramond fields.

In this letter we will generalize the analysis we started in I and prove that the phase transition in our model survives quantum fluctuations and that the quantum loop corrections to the sub-leading order lead to mass and wave function renormalizations. Similar prove was given for the phase transition in rigid QED [3]

The gauge fixed action of the rigid string [2] coupled to the rank two antisymmetric Kalb-Ramond tensor field ϕ is [4]:

$$S_{gauge-fixed} = \mu_0 \int d^2\xi\rho + \frac{1}{2t_0}\int d^2\xi[\rho^{-1}(\partial^2 x)^2 + \lambda^{ab}(\partial_a x\partial_b x - \rho\delta_{ab})] + S_{K-R} \quad (1a)$$

where

$$S_{K-R} = e_0 \int d^2\xi \epsilon^{ab} \partial_a x^\mu \partial_b x^\nu \phi_{\mu\nu} + \frac{1}{12} \int d^4x F_{\mu\nu\rho} F^{\mu\nu\rho} \ . \qquad (1b)$$

where e_0 is a coupling constant of dimension $length^{-1}$, t_0 is the bare curvature coupling constant which is dimensionless and F is the Abelian field strength of ϕ. The integration of the ϕ field is Gaussian. We obtain the following interacting long range Coulomb-like term that modifies the rigid string:

$$\frac{1}{2t_0} \int \int d^2\xi d^2\xi' \sigma^{\mu\nu}(\xi) \sigma_{\mu\nu}(\xi') V(|x - x'|, a) \qquad (1c)$$

where V is the analogue of the long range Coulomb potential:

$$V(|x - x'|, a) = \frac{2g_0}{\pi} \frac{1}{|x(\xi) - x(\xi')|^2 + a^2\rho} \ . \qquad (1d)$$

where $\sigma^{\mu\nu}(\xi) = \epsilon^{ab} \partial_a x^\mu \partial_b x^\nu$. We have introduced the cut-off "a" to avoid the singularity at $\xi = \xi'$ and define $g_0 = t_0 \alpha_{Coulomb} = t_0 \frac{e_0^2}{4\pi}$ which has dimension of $length^{-2}$. The partition function is

$$Z = \int D\lambda D\rho Dx exp(-S_{eff}) \ . \qquad (2)$$

where the effective action S_{eff} is (1a) and (1c).

The long range KR interactions are non-local and impossible to integrate. Therefore we consider

$$x^\nu(\xi) = x_0^\nu(\xi) + x_1^\nu(\xi)$$

and expand the Coulomb term (1c,d) to quadratic order in $x_1^\nu(\xi)$ about the background straight line x_0. The x-integration is now Gaussian and to the leading D approximation we obtain the new effective action S_{eff}:

$$S_{0eff} = \frac{1}{2t_0} [\int d^2\xi (\lambda^{ab}(-\rho\delta_{ab}) + 2t_0\mu_0\rho) + t_0 D tr ln A] \qquad (3)$$

where A is the operator

$$A = \partial^2 \rho^{-1} \partial^2 - \partial_a \lambda^{ab} \partial_b + V(\xi, \xi') \ . \qquad (4)$$

In the large D limit the stationary point equations resulting from varying λ and ρ

respectively are:

$$\rho = \frac{t_0 D}{2} trG \tag{5a}$$

$$2t_0\mu_0 - \lambda^{ab}\delta_{ab} = t_0 Dtr(\rho^{-2}(-\partial^2 G)) \tag{5b}$$

where the world sheet Green's function is defined by:

$$G(\xi,\xi') =< \xi|(-\partial^2)A^{-1}|\xi' > \tag{6}$$

The stationary points are:

$$\rho(\xi) = \rho^*, \qquad \lambda^{ab} = \lambda^* \delta^{ab} \tag{7}$$

where ρ^* and λ^* are constants. Using the stationary solutions (7) the operator A is given by:

$$trlnA = A_\Sigma \int \frac{d^2p}{(2\pi)^2} ln[p^4 + p^2m^2 + p^2 V_0(p) + V_1(p)] \tag{8}$$

where A_Σ is the area of the surface and

$$V_0(p) = \frac{4g_0\rho^*}{\pi} \int d^2\xi \frac{e^{ip.\xi}}{\xi^2 + a^2} = 8g_0\rho^* K_0(a|p|) \tag{9a}$$

$$V_1(p) = \frac{8g_0\rho^*}{\pi} \int d^2\xi \frac{[e^{ip.\xi} - 1]}{(\xi^2 + a^2)^2} = \frac{8g_0\rho^*}{a^2}(a|p|K_1(a|p|) - 1) \tag{9b}$$

where $K_n(z), n = 0, 1, ..$ is the Bessel function of the third kind and

$$K_1(z) := -\frac{d}{dz} K_0(z) \ . \tag{10}$$

Thus (5a) becomes the mass gap equation [2]:

$$1 = \frac{Dt_0}{2} \int \frac{d^2p}{(2\pi)^2} \frac{p^2}{p^2(p^2 + m_0^2) + p^2 V_0(p) + V_1(p)} \tag{11}$$

where we have defined the bare mass:

$$m_0^2 = \rho^* \lambda^* \tag{12}$$

and it is associated with the propagator:

$$< \partial_a x^\mu(p) \partial_a x^\nu(-p) >= \frac{Dt_0}{2} \frac{\delta^{\mu\nu}}{p^2 + m_0^2 + \mathcal{V}(p^2)} \tag{13}$$

where

$$\mathcal{V}(p^2) = V_0(a^2p^2) + \frac{V_1(a^2p^2)}{p^2} \ . \tag{14}$$

On the other hand eq(5b) yields the string tension renormalization condition:

$$\mu_0 = \frac{D}{8\pi} \frac{\Lambda^2}{\rho^*} - \frac{D}{2\rho^*} \int \frac{d^2p}{(2\pi)^2} \frac{\mathcal{V}(p^2)}{p^2 + m_0^2 + \mathcal{V}(p^2)} \tag{15}$$

where $\Lambda = \frac{1}{a}$ is an U.V. cut-off. To obtain a non-zero phase transition temperature the mass gap equation must be infra-red finite for $m_0^2 = 0$. Therefore without the K-R long range interactions ($g_0 = 0$) the theory exists only in the disordered phase $t_0 > t_c$ and the U.V stable fixed point is $t_c = 0$. In this case the beta function of the free rigid string theory is asymptotically free, indicating the absence of the extrinsic curvature term at large distance scales. In contrast to the naive classical limit the theory is therefore well behaved and free of ghosts. As we have shown in I this property prevails in the sub-leading quantum corrections to the mass gap equation. The ordered and disordered phases are separated by the critical line defined by eq.(11) at $m_0^2 = 0$:

$$1 = \frac{Dt_0}{4\pi} \int_0^\epsilon dy \frac{y^3}{y^4 + \kappa_0(y^2 K_0(y) + yK_1(y) - 1)} \tag{16}$$

where $\kappa_0 = 8g_0\rho^* a^2$ is a dimensionless coupling constant. We have made the change of variable y=ap and introduced the U.V cut-off Λ, and $\epsilon = \frac{\Lambda}{\Lambda_0}$ where $\Lambda_0 = \frac{1}{a}$. It is straightforward to prove that there exist an κ^* (c.f. Fig.1) for which any choice of ϵ leads to phase transition as long as $\kappa_0 < \kappa^*$. We set $\epsilon = 1$. It is remarkable

that eq.(16) is finite except at $\kappa_0 = 0$ (absence of K-R interactions). In fact after tedious calculations one can prove that

$$lim_{y\to 0}[\frac{y^3}{y^4 + \kappa_0(y^2 K_0(y) + yK_1(y) - 1)}] = 0$$

thus having an infra-red finiteness.

The dimensionless coupling constant $\kappa_0 = 8g_0\rho^* a^2$ has a natural interpretation. From the string renormalization condition (15) one obtains the critical value of the string tension μ_c to be

$$\mu_c = \frac{D}{4\pi a^2 \rho^*} \int_0^1 dy \frac{y^5}{y^4 + \kappa_0(y^2 K_0(y) + yK_1(y) - 1)}$$

i.e

$$\mu_c := \frac{D}{8\pi a^2 \rho^*} f(\kappa_0)$$

where f is a positive function of κ_0. Combing the above result with the definition of κ_0 we finally obtain:

$$\kappa_0 = h(\frac{Dg_0}{\pi \mu_c})$$

as a positive function of the ratio of the Kalb-Ramond coupling constant to the critical string tension.

The critical curve distinguishing the two phases in the (t_0, κ_0) plane is shown in Fig.1. The order parameter of the theory is the mass gap equation (15) where m_0^2 is the parameter that distinguishes that two phases. In the disordered phase $m_0^2 > 0$ and $< \partial x_c^0 >= 0$, while in the ordered phase it is straightforward to show that $m_0^2 = 0$ and the order parameter is the analogue of the classical magnetization, namely $< (\partial x_c^0)^2 >$, thus the mass gap is

$$< (\partial x_c^0)^2 >= 1 - \frac{Dt_0}{2} \int \frac{d^2 p}{(2\pi)^2} \frac{1}{p^2 + \mathcal{V}(p^2)}.$$

In the disordered phase the coupling constants t_0 and κ_0 are completely fixed by dimensional transmutation in terms of the cut-off Λ^2 and m_0^2. Thus, they cannot be fine tuned. This is an important property that is vital in proving the absence of ghosts in our model [5].

II-The Loop Corrected Mass Gap Equation

In mean field theory i.e leading order in $\frac{1}{D}$, the relevant propagator is equation (13). In the sub-leading correction to mean field theory, the quantum fluctuation imply a new term corresponding to the self-energy of the $\partial_a x^\mu$-field

$$< \partial_a x^\mu(p)\partial_a x^\nu(-p) >= \frac{Dt_0}{2}\frac{\delta^{\mu\nu}}{(p^2+m_0^2+\mathcal{V}(p^2)+\frac{1}{D}\Sigma(p))} \ . \tag{17}$$

The new contribution $\Sigma(p)$ arises from fluctuations of the Lagrange multipliers λ_{ab} and ρ where the fluctuations σ_{ab} and η are defined by:

$$\lambda_{ab} = \lambda^*\delta_{ab} + i\frac{1}{\sqrt{(D/2)}}\sigma_{ab}$$

$$\rho = \rho^*(1 + i\frac{\rho^*}{\sqrt{(D/2)}}\eta) \ . \tag{18}$$

Expanding the effective action (3) in powers of σ_{ab} and η, it is straightforward to extract the σ_{ab} and η propagators (Fig (2)):

$$\Pi_{ab|cd}(p) = \frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})\pi(p^2) \tag{19a}$$

$$\pi(p^2) = \int \frac{d^2k}{(2\pi)^2}\frac{1}{(k^2+m_0^2+\mathcal{V}(k^2))((p+k)^2+m_0^2+\mathcal{V}((p+k)^2))} \tag{19b}$$

$$\tilde{\pi}(p^2) = 2\int \frac{d^2k}{(2\pi)^2}\frac{(k.(k+p))^2}{(k^2+m_0^2+\mathcal{V}(k^2))((p+k)^2+m_0^2+\mathcal{V}((p+k)^2))} \ . \tag{19c}$$

It is obvious from (19c) that the η propagator has quadratic and logarthmic divergences and therefore needs regularization before attempting to analyze the self energy. These type of divergences were exactly calculated in I, and therefore can be subtracted in a similar fashion here. The only exception is that we cannot find an exact analytical answer for both π and the regularized π^* because of the complexity and the non-polynomial structure of the potential $\mathcal{V}(p^2)$. Fortunately, we do not need the exact analytical forms of the π's because all we are interested in is the ultra-violet behaviour of the self energy Σ from which we can extract the divergent

contributions. For this we only need the asymptotic forms of the propagators and they are:

$$\pi_{asy}(p^2) = \frac{1}{2\pi(p^2+\mathcal{V}(p^2))} log\frac{p^2}{m_0^2} \tag{20a}$$

$$\pi^*{}_{asy}(p^2) = \frac{p^2+\mathcal{V}(p^2)}{4\pi} log\frac{p^2}{m_0^2} \ . \tag{20b}$$

The self energy Σ can be computed from the diagrams of Fig(3). These diagrams are of order $\frac{1}{D}$ and represent the quantum fluctuations:

$$\Sigma(p) = \int \frac{d^2k}{(2\pi)^2} \frac{\pi^{-1}(k^2)}{((p+k)^2+m_0^2+\mathcal{V}((p+k)^2))}$$

$$+\frac{1}{2}\int \frac{d^2k}{(2\pi)^2} \frac{\pi^{*-1}(k^2)(k.(k+p))^2}{((p+k)^2+m_0^2+\mathcal{V}((p+k)^2))}$$

$$-\int \frac{d^2k}{(2\pi)^2}\int \frac{d^2q}{2\pi} \frac{\pi^{-1}(0)}{(q^2+m_0^2+\mathcal{V}(q^2))^2} \frac{\pi^{-1}(k^2)}{((q+k)^2+m_0^2+\mathcal{V}((q+k)^2))}$$

$$-\frac{1}{2}\int \frac{d^2k}{(2\pi)^2}\int \frac{d^2q}{2\pi} \frac{\pi^{-1}(0)}{(q^2+m_0^2+\mathcal{V}(q^2))^2} \frac{\pi^{*-1}(k^2)(k.(k+q))^2}{((q+k)^2+m_0^2+\mathcal{V}((q+k)^2))} \ . \tag{21}$$

A Taylor expansion of the self energy about zero momentum leads to mass and wave function renormalizations and a remaining piece $\tilde{\Sigma}$ which must be finite for the theory to be renormalizable. The propagator now reads:

$$\frac{Z}{(p^2+m^2+\mathcal{V}(p^2)+\frac{1}{D}\tilde{\Sigma}_{finite}(p))} \ . \tag{22}$$

where

$$Z = 1 - \frac{1}{D}\Sigma'(0) \tag{23a}$$

is the wave function renormalization,

$$m^2 = m_0^2 + \frac{1}{D}(\Sigma(0) - m_0^2\Sigma'(0)) \tag{23b}$$

is mass renormalization and

$$\kappa = Z\kappa_0 \tag{23c}$$

is the K-R coupling renormalization. The renormalized mass gap equation is:

$$1 = \frac{Dt}{2}\int \frac{d^2p}{(2\pi)^2}\frac{1}{(p^2 + m^2 + \mathcal{V}(p^2) + \frac{1}{D}\tilde{\Sigma}_{finite}(p))}\ . \tag{24}$$

To examine whether there is still a phase transition i.e the infra-red finiteness of the renormalized mass gap equation eq.(24) at $m^2 = 0$, we must examine the renormalizability of the theory. We have seen in the examples of rigid QED [3] and the free rigid string in part I that all typical divergences come from $\Sigma(0)$ and $\Sigma'(0)$ which can be absorbed in mass and wave function renormalizations. In order to calculate the finite regularized self energy we need to simplify further Eq.(21). Using (19) we can replace zero-momentum insertions of the σ and ρ fields and rewrite (21) as:

$$\Sigma(p) = \Sigma_1(p) + \frac{1}{2}I_1\int \frac{d^2k}{(2\pi)^2}\pi^{*-1}(k^2) - \frac{1}{2}\int \frac{d^2k}{(2\pi)^2}k^2\pi^{*-1}(k^2) \tag{25}$$

$$+\int \frac{d^2k}{(2\pi)^2}\frac{\pi^{-1}(k^2)}{((p+k)^2 + m_0^2 + \mathcal{V}((p+k)^2))} + \frac{1}{2}\pi(0)^{-1}\int \frac{d^2k}{(2\pi)^2}\pi^{-1}(k^2)\frac{\partial}{m_0^2}\pi(k^2)$$

$$+\frac{1}{2}\int \frac{d^2k}{(2\pi)^2}\frac{\pi^{*-1}(k^2)(k.(k+p))^2}{((p+k)^2 + m_0^2 + \mathcal{V}((p+k)^2))} + \frac{1}{4}\pi(0)^{-1}\int \frac{d^2k}{(2\pi)^2}\pi^{*-1}(k^2)\frac{\partial}{m_0^2}\pi^*(k^2)$$

where $\Sigma_1(p)$ is a finite piece of the self energy and

$$I_1(\frac{\Lambda^2}{m_0^2}, \kappa_0) = \pi(0)^{-1}\int \frac{d^2q}{(2\pi)^2}\frac{1}{(q^2 + m_0^2 + \mathcal{V}(q^2))} = \frac{2\pi(0)^{-1}}{Dt_0}\ . \tag{26}$$

where we have used the mass gap eq. (11). In deriving (25) we have used the facts that $lim_{p\to\infty}\frac{\mathcal{V}(p^2)}{p^n} = 0, \quad n \geq -1$ which follows from the asymptotic properties of the Bessel functions of the third kind.

As in part I we can now regularize (25) by applying the SM regularization scheme [4] where one subtracts the highest powers of the integration variable appearing in the Taylor expansion of the integrands in (25). Again the divergences in (25) occur only at the zeroth and first order expansions. Using the asymptotic limits of the propagators (19) we finally obtain the following finite regularized self energy:

$$\Sigma_{finite}(p) = \Sigma(p) + I_1(1 - \frac{1}{2}I_0) - \frac{m_0^2}{4}I_0 - \frac{1}{4}(p^2 + m_0^2)I_0 \tag{27a}$$

where

$$I_0(\frac{\Lambda^2}{m_0^2}, \kappa_0) = \int \frac{d^2k}{(2\pi)^2} \frac{4\pi}{(k^2 + \mathcal{V}(k^2))log\frac{k^2}{m_0^2}} \ . \tag{27b}$$

The results (26) and (27) generalize equation (23) in part I for the free rigid string to the interacting rigid string with long range K-R fields.

We can now read from (27) and (23c) the mass, wave function, and K-R renormalizations:

$$m^2 = m_0^2[1 - \frac{1}{D}(\frac{I_1}{m_0^2}(1 - \frac{1}{2}I_0) - \frac{1}{4}I_0)] \tag{28a}$$

$$Z = 1 - \frac{1}{4D}I_0 \ . \tag{28b}$$

The beta function can now be obtained as in part I by holding $m^2(\Lambda, m_0, t)$ fixed:

$$\beta(t) = -(\frac{\partial logm^2}{\partial t})^{-1} \ . \tag{29}$$

Having shown that the theory is renormalizable we can now give the renormalized critical line defined by (24) at $m^2 = 0$

$$1 = \frac{Dt}{2} \int \frac{d^2p}{(2\pi)^2} \frac{1}{(p^2 + \mathcal{V}(p^2) + \frac{1}{D}\tilde{\Sigma}_{finite}(p))} \tag{30}$$

which is certainly is Infra-red finite since by its definition $\tilde{\Sigma}_{finite}(0) = 0$ and (30) then has the exact behaviour as (16) at y=0. Furthermore, as long as $\kappa < \kappa^*$ we have shown that the critical line (16) has no real poles. i.e $(p^2 + \mathcal{V}(p^2)) > 0$. The presence of $\tilde{\Sigma}_{finite}$ will not affect the above conclusion to any sub-leading finite

order in perturbation theory because in the sub-leading $\frac{1}{D}$ order the critical line is:

$$1 = \frac{Dt}{2} \int \frac{d^2p}{(2\pi)^2} \left(\frac{1}{(p^2 + \mathcal{V}(p^2))} - \frac{1}{(p^2 + \mathcal{V}(p^2))} \frac{1}{D} \tilde{\Sigma}_{finite}(p) \frac{1}{(p^2 + \mathcal{V}(p^2))} \right) . \quad (31)$$

As in any quantum field theory the location of the poles in the presence of $\tilde{\Sigma}_{finite}(p)$ is a non-perturbative issue and requires the form of $\tilde{\Sigma}_{finite}(p)$ to all orders in perturbation theory.

Acknowledgement I am very grateful to Prof. A. Polyakov for his constant encouragement and extensive support over the last year and a half and for suggesting that we address the quantum stability of the phase transition both in the model of rigid QED and that of rigid strings coupled to long range interactions [6]. I am also grateful to Prof. Y. Nambu for his extensive support, encouragement and long discussions over the last two years without which we could not have gone far in our investigations. I also thank P.Ramond, C. Thorn, and Z. Qui for constructive discussions and suggestions. Finally my gratitude to my friend D. Zoller for a fruitful and devoted collaboration over the years.

References

[1] A. Polyakov, Nucl. Phys. B268 (1986) 406 ; A. Polyakov, Gauge fields, and Strings, Vol.3, harwood academic publishers

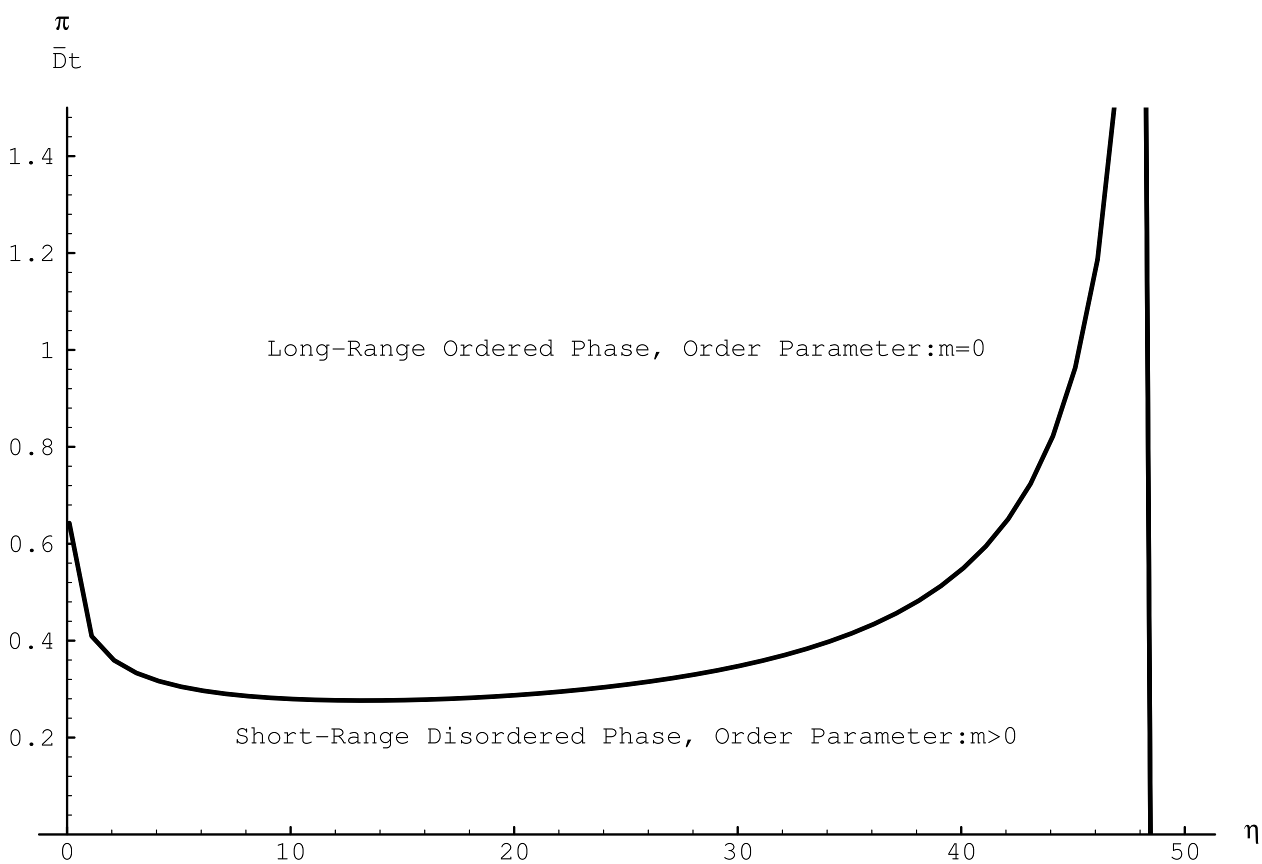
[2] M. Awada and D. Zoller, Phys.Lett B325 (1994) 115

[3] M. Awada, D. Zoller, and J. Clark, Cincinnati preprint June -1 -(1994)

[4] M. Kalb, and P. Ramond Phys. Rev. D Vol.9 (1974) 2237

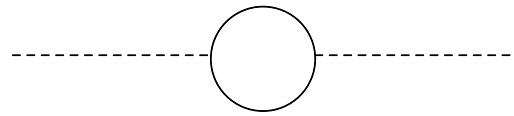
[5] M. Awada and D. Zoller, Phys.Lett B325 (1994) 119

[6] A. Polyakov, Gauge fields, and Strings, Vol.3, harwood academic publishers, J.Orloff and R.Brout, Nucl. Phys. B270 [FS16],273 (1986), M. Campostrini and P.Rossi, Phys. Rev.D 45, 618 (1992) ; 46, 2741 (1992), H. Flyvberg, Nucl. Phys. B 348, 714, (1991).

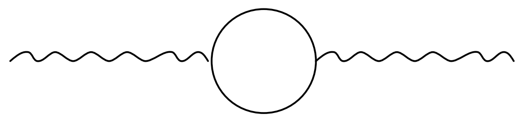


$\frac{\pi}{\bar{D}t}$
1.4
1.2
1
0.8
0.6
0.4
0.2
Long-Range Ordered Phase, Order Parameter:m=0
Short-Range Disordered Phase, Order Parameter:m>0
0
10
20
30
40
50
η

$\tilde{\Pi}(p^2) =$

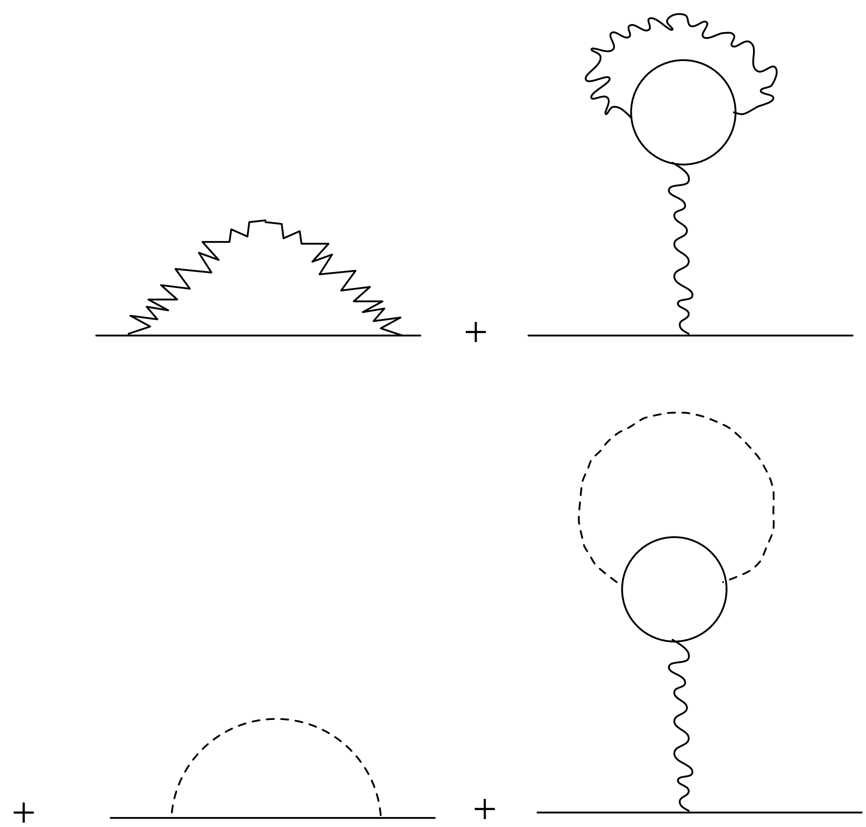


$\tilde{\Pi}_{ab|cd}(p^2) =$



Fig(2)

$\Sigma(p) \quad =$



Fig(3)